

Find all of the zeros and write a linear factorization of the function

28)  $f(x) = x^3 - 10x^2 + 44x - 69$

$$\begin{array}{r|rrrr} 3 & 1 & -10 & 44 & -69 \\ & & 3 & -21 & 69 \\ \hline & 1 & -7 & 23 & 0 \end{array}$$

$$x^2 - 7x + 23 = 0$$

Zeros:

$$x = 3, \frac{7 + i\sqrt{43}}{2}, \frac{7 - i\sqrt{43}}{2}$$

$$(x-3)(x^2 - 7x + 23)$$

$$\begin{aligned} & \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \\ & \frac{7}{2(1)} \pm \frac{\sqrt{(-7)^2 - 4(1)(23)}}{2(1)} \\ & \frac{7}{2} \pm \frac{\sqrt{49 - 92}}{2} \\ & \frac{7}{2} \pm \frac{\sqrt{-43}}{2} \\ & \frac{7}{2} \pm \frac{i\sqrt{43}}{2} \end{aligned}$$

A)  $f(x) = x^5 - 3x^4 - 5x^3 + 5x^2 - 6x + 8$

$$\begin{array}{r|rrrrrr} 1 & 1 & -3 & -5 & 5 & -6 & 8 \\ & & 1 & -2 & -7 & -2 & -8 \\ \hline -2 & 1 & -2 & -7 & -2 & -8 & 0 \\ & & -2 & 8 & -2 & 8 & \\ \hline 4 & 1 & -4 & 1 & -4 & 0 & \\ & & 4 & 0 & 4 & & \\ \hline & 1 & 0 & 1 & 0 & & 0 \end{array}$$

$$x^2 + 1 = 0$$

$$x^2 = -1$$

$$x = \pm\sqrt{-1} = \pm i$$

Zeros:

$$x = 1, -2, 4, \pm i$$

$$(x-1)(x+2)(x-4)(x^2+1)$$

Find all of the zeros and write a linear factorization of the function

$$y = 3x^4 + 4x^3 + 2x^2 - x - 2$$

$$\frac{2}{3}$$

$$\begin{array}{r|rrrrr}
 -1 & 3 & 4 & 2 & -1 & -2 \\
 & & -3 & -1 & -1 & 2 \\
 \hline
 \frac{2}{3} & 3 & 1 & 1 & -2 & 0 \\
 & & 2 & 2 & 2 & \\
 \hline
 & 3 & 3 & 3 & 0 & 
 \end{array}$$

$$\left(\frac{2}{3}\right)^3$$

$$-\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$-\frac{1}{2} \pm \frac{\sqrt{1^2 - 4(1)(1)}}{2}$$

$$-\frac{1}{2} \pm \frac{i\sqrt{3}}{2}$$

$$\frac{3x^2}{3} + \frac{3x}{3} + \frac{3}{3} = \frac{0}{3}$$

$$x^2 + x + 1 = 0$$

$$x^2 + x + \frac{1}{4} = -1 + \frac{1}{4}$$

$$\sqrt{\left(x + \frac{1}{2}\right)^2} = \sqrt{-\frac{3}{4}}$$

$$x + \frac{1}{2} = \pm \frac{i\sqrt{3}}{2}$$

$$x = -\frac{1}{2} \pm \frac{i\sqrt{3}}{2}$$

Zeros

$$x = -1, \frac{2}{3}, -\frac{1}{2} \pm \frac{i\sqrt{3}}{2}$$

$$(x+1)(3x-2)(x^2+x+1) = y$$

Using the given zero find all of the zeros and write a linear factorization

33)  $f(x) = x^4 - 2x^3 - x^2 + 6x - 6$  zero:  $1+i$

$$\begin{array}{r|rrrrr}
 1+i & 1 & -2 & -1 & +6 & -6 \\
 & & 1+i & -2 & -3-3i & 6 \\
 \hline
 1-i & 1 & -1+i & -3 & 3-3i & 0 \\
 & & 1-i & 0 & -3+3i & \\
 \hline
 & 1 & 0 & -3 & 0 & 
 \end{array}$$

$$x^2 - 3 = 0$$

$$x^2 = 3$$

$$x = \pm\sqrt{3}$$

$$\begin{aligned}
 &(1+i)(-1+i) \\
 &-1 + i - i + i^2 \\
 &-1 - 1
 \end{aligned}$$

$$\begin{aligned}
 &(1+i)(3-3i) \\
 &3 - 3i + 3i - 3i^2 \\
 &3 - 3(-1)
 \end{aligned}$$

$$(x-\sqrt{3})(x+\sqrt{3})(x-(1-i))(x-(1+i))$$

Write the function as a product of linear and irreducible quadratic factors all with real coefficients.

42)  $f(x) = x^4 - 2x^3 + x^2 - 8x - 12$

$$\begin{array}{r|rrrrr}
 -1 & 1 & -2 & 1 & -8 & -12 \\
 & & -1 & 3 & -4 & 12 \\
 \hline
 3 & 1 & -3 & 4 & -12 & 0 \\
 & & 3 & 0 & 12 & \\
 \hline
 & 1 & 0 & 4 & 0 & 
 \end{array}$$

$$x^2 + 4$$

$$y = (x+1)(x-3)(x^2+4)$$

Find All possible Zeros

$$f(x) = 3x^5 - 2x^4 + 6x^3 - 4x^2 - 24x + 16$$

Zero  $\sqrt{2}$   $-\sqrt{2}$

$\sqrt{2}$	3	-2	6	-4	-24	16
		$3\sqrt{2}$	$6-2\sqrt{2}$	$-4+12\sqrt{2}$	$24-8\sqrt{2}$	-16
$-\sqrt{2}$	3	$-2+3\sqrt{2}$	$12-2\sqrt{2}$	$-8+12\sqrt{2}$	$-8\sqrt{2}$	0
		$-3\sqrt{2}$	$+2\sqrt{2}$	$-12\sqrt{2}$	$8\sqrt{2}$	
$\frac{2}{3}$	3	-2	12	-8		0
		2	0	8		
	3	0	12	0		

$$\begin{aligned} &\sqrt{2}(-2+3\sqrt{2}) \\ &\quad -2\sqrt{2}+6 \\ &\sqrt{2}(12-2\sqrt{2}) \\ &\quad 12\sqrt{2}-4 \\ &\sqrt{2}(-8+12\sqrt{2}) \\ &\quad -8\sqrt{2}+24 \end{aligned}$$

$$\begin{aligned} 3x^2+12 &= 0 \\ x^2+4 &= 0 \\ x^2 &= -4 \\ x &= \pm 2i \end{aligned}$$

Zeros

$$x = \frac{2}{3}, \sqrt{2}, -\sqrt{2}, 2i, -2i$$